Soliton solution of continuum magnetization equation in a conducting ferromagnet with a spin-polarized current

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Exact soliton solutions of a modified Landau-Lifshitz equation for the magnetization of conducting ferromagnet in the presence of a spin-polarized current are obtained by means of inverse scattering transformation. From the analytical solution the effects of spin current on the frequency, wave number, and dispersion law of spin wave are investigated. The one-soliton solution indicates obviously current-driven precession and periodic shape variation as well. The inelastic collision of solitons, by which we mean the shape change before and after collision, appears due to the spin current. We, moreover, show that complete inelastic collisions can be achieved by adjusting spectrum and current parameters. This may lead to an potential technique for shape control of spin wave.

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Considerable attention has been paid to the dynamics of magnetization associated with spin-polarized current in layered materials and in Mn oxides recently. Both theoretical and experimental investigations mainly concentrated on large magnetoresistance are of fundamental importance in the understanding of magnetism and applied interest in the fabrication of magnetic devices. Spin transfer from spin-polarized current to magnetization of conducting ferromagnetic films is one of the intriguing features which is theoretically proposed in Refs. [1,2] and subsequently verified [4,5] in experiment. Many studies [6–10] on this phenomenon have been followed since the spin-transfer mechanism was first explained conceptually. However, the dynamics of magnetization in the presence of spin current has not been well understood. In Ref. [5] a continuum equation for the magnetization of conducting ferromagnet in the presence of spin-polarized current is derived and is seen to be a modified Landau-Lifshitz equation with an additional topological term. The spatial dependence of the magnetization is evaluated [5] and several solutions of one dimension are also discussed. We in this paper study the soliton solution of the modified Landau-Lifshitz equation given in Ref. [5] from which the current induced precession of the magnetization and soliton-soliton collisions are investigated. The effect of spin current on the dynamics of magnetization is demonstrated explicitly. We obtain exact soliton solutions by means of inverse scattering transformation in one-dimensional geometry. An intriguing feature is that inelastic collisions generally appear due to the spin-polarized current and the complete inelastic collisions which may lead to a interesting technique of soliton filter and switch can be achieved in special cases by adjusting the parameters spin-polarized current.

Following Ref. [5] we consider a current propagating through conducting ferromagnet and assume that the conducting electrons interact only with the local magnetization **M**. A continuum equation for the magnetization is obtained in the local magnetization frame as a Landau-Lifschitz type [5],

$$\frac{\partial}{\partial t}\mathbf{M} = \mathbf{M} \times \tilde{J}\mathbf{M}_{zz} - \gamma \mathbf{M}_z, \tag{1}$$

where $\mathbf{M}(z,t)=(M^x(z,t),\, M^y(z,t),\, M^z(z,t))$ is the local magnetization and $\tilde{J}=(g\mu_B M/\hbar)J$ with J being the exchange interacting constant between the local magnets. The parameter γ describes effect of the current which is the same as defined in Ref. [5]. The length of magnetization vector is set to unit, $\mathbf{M}^2(z,t)=1$, for the sake of simplicity. We begin with the inverse scattering transformation which is a useful method to solve the nonlinear Eq. (1). By means of the Ablowitz-Kaup-Newell-Segur method one can construct Lax equations for the Eq. (1) as

$$\begin{aligned}\frac{\partial}{\partial z}\Psi(z,t,\lambda) &= U_1(\lambda)\Psi(z,t,\lambda),\\ \frac{\partial}{\partial t}\Psi(z,t,\lambda) &= U_2(\lambda)\Psi(z,t,\lambda),\end{aligned} \tag{2}$$

where λ is a spectrum parameter, $\Psi(z,t,\lambda)$ is eigenfunction corresponding to the spectrum λ. The operators $U_1(\lambda)$ and $U_2(\lambda)$ are given in the following form:

$$\begin{aligned}U_1 &= -i\tilde{J}(\lambda-\lambda_0)(M\sigma),\\ U_2 &= i2\tilde{J}^3(\lambda^2-\lambda_0^2)(M\sigma) - \tilde{J}^2(\lambda-\lambda_0)(M\sigma)(M_z\sigma),\end{aligned} \tag{3}$$

where the real parameter $\lambda_0=\gamma(4\tilde{J}^2)^{-1}$ indicates the effect of the current and σ is Pauli matrix. Thus Eq. (1) can be recovered from the compatibility condition $(\partial/\partial t)U_1-(\partial/\partial z)U_2+[U_1,U_2]=0$. We consider the following natural boundary condition at initial time $(t=0)$, $\mathbf{M}(z)\equiv(M^x,M^y,M^z)\to(0,0,1)$ as $|z|\to\infty$. We then have the asymptotic form of Eq. (2) at $|z|\to\infty$

$$\partial_z E(z,\lambda) = L_0(\lambda) E(z,\lambda), \tag{4}$$

where

$$E(z,\lambda) = e^{-i\tilde{J}(\lambda-\lambda_0)z\sigma_3}, \quad L_0(\lambda) = -i\tilde{J}(\lambda-\lambda_0)\sigma_3. \tag{5}$$

Based on the Lax Eqs. (2), we can derive the exact solution of N-soliton trains by employing of the inverse scattering transformation [11,12]. As a special case that $N=1$ the exact one-soliton solution is given as follows:

$$M^x = \frac{1}{\Delta_1}[-2(\alpha_1-\lambda_0)\beta_1 \sin(\Phi_1-\phi_1)\cosh\Theta_1 - 2\beta_1^2\cos(\Phi_1-\phi_1)\sinh\Theta_1],$$

$$M^y = \frac{1}{\Delta_1}[2(\alpha_1-\lambda_0)\beta_1 \cos(\Phi_1-\phi_1)\cosh\Theta_1 - 2\beta_1^2\sin(\Phi_1-\phi_1)\sinh\Theta_1]$$

$$M^z = 1-\frac{2\beta_1^2}{\Delta_1}, \tag{6}$$

where

$$\Delta_1 = |\lambda_1-\lambda_0|^2\cosh^2\Theta_1,$$

$$\Theta_1 = 2\tilde{J}\beta_1(z-V_{1,M}t)-z_1,$$

$$\Phi_1 = 2\tilde{J}(\alpha_1-\lambda_0)z - 4\tilde{J}^3[\alpha_1^2-\beta_1^2-\lambda_0^2]t-\phi_1.$$

Parameter $V_{1,M}=4\tilde{J}^2\alpha_1$ denotes the velocity of envelope, $z_1=\ln[(2\tilde{J}\beta_1)^{-1}c_1]$ is the center position, and $\phi_1=\arg[\tilde{J}(\lambda_1-\lambda_0)]$ is the initial phase of the spin wave. The parameter $\lambda_1=\alpha_1+i\beta_1$ denotes eigenvalue with α_1,β_1 being the real and imaginary parts, respectively, and c_1 is a real constant of integration. The solution Eq. (6) describes a current-driven precession of magnetization with periodic shape variation. The center of solitary wave moves with velocity $V_{1,M}$, while the wave amplitude and width vary periodically with time. We see that the spin-polarized current imparts a torque to the magnetization due to local exchange interaction between electron spin and the magnetic moment. This observation is in accord with the prediction in Refs. [1–3,5,9]. As a consequence of reaction the current flow is strongly affected by the orientation of the magnetic moments. Thus a higher electrical resistance in magnetic layer may occur. The spin-polarized current can be used to adjust the precession of magnetic moment and the wave shape as well. We then provide in principle a mechanism of current control of the spin wave.

To see closely the physical significance of one-soliton solution it is helpful to show the parameter dependence of Euler angles of the magnetization vector which in a spherical coordinate is written as $\mathbf{M}(z,t)\equiv(\sin\theta\cos\varphi,\sin\theta\sin\varphi,\cos\theta)$. From the Eq. (6) we find

$$\cos\theta = 1-\frac{A_M}{\cosh^2[\mathcal{F}_1^{-1}(z-V_{1,M}t)-z_1]}, \tag{7}$$

$$\varphi = \frac{\pi}{2}-\phi_1+k_1 z-\Omega_1 t+\arctan\left(\frac{\beta_1}{\alpha_1-\lambda_0}\tanh\Theta_1\right), \tag{8}$$

where $A_M=2\beta_1^2/|\lambda_1-\lambda_0|^2$, $\mathcal{F}_1=1/2\tilde{J}\beta_1$ are amplitude and width of the soliton, respectively. The wave number is $k_1=k_0-k_S$ with $k_0=2\tilde{J}\alpha_1$ denoting the wave number in the absence of the current while $k_S=2\tilde{J}\lambda_0$ is the wave number shift induced by the spin-polarized current. The frequency of magnetization precession is seen to be $\Omega_1=\Omega_0-\Omega_S$ with $\Omega_0=4\tilde{J}^3(\alpha_1^2-\beta_1^2)$ being the frequency in the absence of current and $\Omega_S=4\tilde{J}^3\lambda_0^2$ the frequency shift induced by spin-polarized current. We see that the effect of current reduces both the wave number and frequency. For a large enough current such that $\lambda_0^2>\alpha_1^2-\beta_1^2$ an instability occurs [4]. We can rewrite the frequency, i.e., the energy spectrum as

$$\Omega_1 = \tilde{J}k_1(k_1+2k_S)-4\tilde{J}^3\beta_1^2. \tag{9}$$

We then see that in the absence of current the minimum of the energy spectrum, i.e., $\Omega_{0,\min}=0$ is located at $k_{0,\min}=\sqrt{4\tilde{J}^2\beta_1^2}$ while the current shifts the position of minimum by an amount $\delta=\sqrt{k_S^2+4\tilde{J}^2\beta_1^2}-(k_S+\sqrt{4\tilde{J}^2\beta_1^2})$. In the absence of spin-polarized current, i.e., $\lambda_0=0$, the solution Eqs. (7) and (8) reduce to the soliton solution in an isotropic spin chain [13].

In the limit case that the amplitude A_M approaches zero, namely $\beta_1\to 0$, the soliton width $\mathcal{F}_1$ diverges, the envelope velocity $V_{1,M}$ attains its maximum value $2\sqrt{\tilde{J}\Omega_0}$, and the solution shown in Eqs. (7) and (8) takes the form such that

$$M^z\to 1, \quad \varphi\to\frac{\pi}{2}-\phi_1+k_1 z-\Omega_1 t,$$

indicating a small linear solution of magnon. In this case the quadratic dispersion law is seen to be $\Omega_1=\tilde{J}(k_0^2-k_S^2)=\tilde{J}k_1(k_1+2k_S)$. We also notice that the phase velocity of the precession is $\Omega_1/k_1=V_{1,M}/2+V_S/2$ which possesses a correction value $V_S/2$ which is half of envelope velocity whereas the group velocity of precession $d\Omega_1/dk_1=V_{1,M}$ coincides with the envelope velocity.

We now consider another special case of the general N-soliton trains, i.e., the two-soliton solution which is seen to be

$$M^x = \mathrm{Re}[-i2\Gamma_2(1-i\Gamma_1)],$$

$$M^y = \mathrm{Im}[i2\Gamma_2(1-i\Gamma_1)],$$

$$M^z = |1-i\Gamma_1|^2-|\Gamma_2|^2, \tag{10}$$

where

$$\Gamma_1 = \frac{1}{W}[(g_1 - g_3)g_6 + (g_2 - g_4)g_5],$$

$$\Gamma_2 = \frac{1}{W}[-(\overline{g_1} - \overline{g_3})g_8 - (\overline{g_2} - \overline{g_4})g_7],$$

with

$$g_1 = 1 + |q_1|^2 + \chi_1\overline{\chi_2}q_1\overline{q_2},$$

$$g_2 = 1 + |q_2|^2 + \overline{\chi_1}\chi_2\overline{q_1}q_2,$$

$$g_3 = \overline{\chi_1}|q_1|^2 + \chi_1 q_1\overline{q_2},$$

$$g_4 = \overline{\chi_2}|q_2|^2 + \chi_2\overline{q_1}q_2,$$

$$g_5 = -\xi_1|q_1|^2 - \chi_1\xi_2 q_1\overline{q_2},$$

$$g_6 = -\xi_2|q_2|^2 - \chi_2\xi_1\overline{q_1}q_2,$$

$$g_7 = -\xi_1\overline{q_1},$$

$$g_8 = -\xi_2\overline{q_2},$$

$$\chi_1 = \frac{2\beta_1(\lambda_1 - \lambda_0)}{-i(\lambda_1 - \overline{\lambda_2})|\lambda_1 - \lambda_0|},$$

$$\chi_2 = \frac{2\beta_2(\lambda_2 - \lambda_0)}{-i(\lambda_2 - \overline{\lambda_1})|\lambda_2 - \lambda_0|},$$

$$W = g_1 g_2 - g_3 g_4,$$

$$q_j = e^{-\Theta_j + i\Phi_j},$$

$$\xi_j = 2\beta_j|\lambda_j - \lambda_0|^{-1}$$

and

$$\Theta_j = 2\tilde{J}\beta_j(z - V_{j,M}t) - z_j,$$

$$\Phi_j = k_j z - \Omega_j t - \phi_j, \qquad (11)$$

where $V_{j,M}=4\tilde{J}^2\alpha_j$ denotes the velocity of envelope, $z_j=\ln[(2\tilde{J}\beta_j)^{-1}c_j]$ the center position, $\phi_j=\arg[\tilde{J}(\lambda_j-\lambda_0)]$ the initial phase, $k_j=2\tilde{J}(\alpha_j-\lambda_0)$ the wave number, and $\Omega_j=4\tilde{J}^3[\alpha_j^2-\beta_j^2-\lambda_0^2]$ is frequency. The parameter $\lambda_j=\alpha_j+i\beta_j$ is the eigenvalue parameter, and c_j is the real constant of integration, $j=1,2$. The solutions (10) describe in general an inelastic scattering process of two solitary waves with different center velocities and different shape-variation frequencies. Before collision, the two solitons move towards each other, one with velocity V_1 and shape variation frequency Ω_1, the other with V_2 and Ω_2, respectively. The interaction potential between two solitons is a complicated function of current-dependent parameter λ_0 and eigenvalue λ_j. For the case that $\alpha_j=\beta_j$, the shape-variation frequencies $\Omega_j(j=1,2)$ of two-soliton depend only on the parameters of spin-polarized current seen from Eq. (11). In the case of $\lambda_0=0$, the solutions (10) reduce to that of the usual two-soliton solution with two center velocities while without shape change [13]. An interesting process in the absence of spin-polarized current is that the collision can result in the interchange of amplitude A_j and phase $\Phi_j(j=1,2)$ exactly as in the case of the elastic collision of two particles [13].

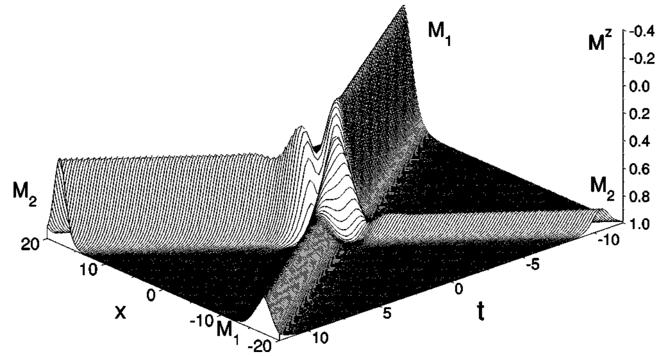

FIG. 1. Inelastic head on collision expressed by Eq. (10) when M_1 is suppressed, where $\lambda_1=-0.45+i0.3$, $\lambda_2=0.52-i0.47$, $\tilde{J}=0.9$, $c_1=0.55$, $c_2=-3.8$, $\lambda_0=0.2$.

It is interesting to show the inelastic collision graphically. The head on collision is explained in Figs. 1 and 2 for suppressed amplitudes of M_1 and M_2, respectively, after collision. This result shows that we may adjust the incoming spin current and the spectral parameters to control the shape of soliton of the magnetization. The dissipationless quantum spin current at room temperature reported in Ref. [10] may be used to realize experimentally the soliton control in the future. Our theoretical observations predict the magnetic random-access memories in which the memory elements are controlled by local exchange-effect forces induced by spin-polaried current rather than by long-range magnetic fields.

In terms of inverse scattering transformation the exact soliton solutions for the magnetization in conducting ferromagnet in the presence of a spin-polarized current are ob-

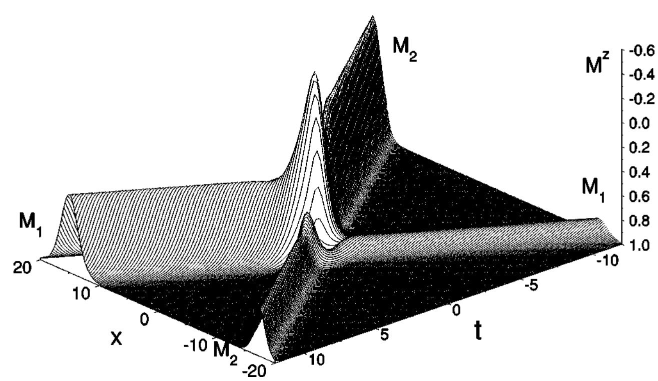

FIG. 2. Inelastic head on collision expressed by Eq. (10) when M_2 is suppressed, where $\lambda_1=0.4-i0.3$, $\lambda_2=-0.5+i0.45$, $\tilde{J}=0.9$, $c_1=-0.5$, $c_2=3.5$, $\lambda_0=0.2$.

tained. Our solutions predict two intrinsic features of the effect of spin-polarized current on the magnetization: (1) Spin-polarized current induces the precession and shape variation of the solitary waves of magnetization. (2) The inelastic collision of solitons. The effect of spin-polarized current on the magnetization is similar to that of the periodically time-varying external magnetic fields reported earlier [11] and is in agreement with the observations in References. [1–3,5,9].

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