Bragg diffraction of interacting Bose-Einstein condensates

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We develop a theoretical model to investigate the Bragg diffraction of an elongated interacting Bose-Einstein condensate which is illuminated by a pair of laser beams. We find that the mean-field effect resulting from the atomic interaction plays an important role in modifying the atomic coherence. Our results show that both the repulsive and attractive interactions would dampen the momentum oscillation of the condensate; they establish surprisingly distinguishable equilibria for the atomic occupations among different diffraction orders. We also give an experimental proposal to observe this phenomenon.

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I. INTRODUCTION

Atom optics is a fast-evolving field with exciting discoveries reported on a daily basis [1]. Among the many highly developed optical elements in atom optics, Bragg diffraction is a versatile technique for manipulating atomic samples [2–7]. With the experimental realization of Bose-Einstein condensates (BECs) in dilute atomic gases [8], a unique platform is provided for us in exploring qualitatively interesting quantum phenomena in various light-atom systems and interaction processes [9–13]. The major advantage of BECs is that essentially all the atoms in the condensate are in perfect coherence, occupying the same quantum state and behaving identically. Thus microscopic quantum phenomena of the atom-field system could be magnified and measured as macroscopic observables. With BECs, light diffraction and scattering have been studied in various contexts, including Bragg spectroscopy [14], condensate coherence analysis [15], matter-wave amplification [16], superfluid-to-Mott insulator phase transition [17], and so on.

In typical matter-wave diffraction experiments, the BECs interact with standing-wave light, which is formed by a pair of countergoing laser beams; matter-wave Bragg diffraction in this case is characterized by periodical oscillations of the momentum distribution of the diffracted matter wave in a time domain, like Pendellösung oscillation. Recently, Li *et al.* [18] performed an experiment in which an elongated BEC was illuminated by a pair of countergoing laser beams of very different intensities. A similar periodical momentum oscillation and matter-wave self-imaging induced by atomic center-of-mass motion were observed. However, with the presence of atomic interaction, the well-defined orders established in the atom-field system would be greatly modified; interesting behaviors of the matter-wave diffraction would be expected in the atom-field system.

In this article, we study the Bragg diffraction of an elongated BEC which is exposed to a pair of laser beams from the same source. The article is organized as follows. In Sec. II, a schematic description of the experimental setup under consideration is provided. We give a theoretical model to describe the coupling of the BEC with electromagnetic fields. Then, in Sec. III, we establish a mean-field treatment for the atomic interaction. In Sec. IV, a detailed discussion is carried out to analyze the different atomic-occupation behaviors at equilibrium for different atomic interactions predicted in Sec. III. Finally, in Sec. V, we give a summary of our results.

II. EXPERIMENTAL SETUP AND THEORETICAL MODEL

We consider an experiment in which an elongated BEC prepared with ^{87}Rb atoms is illuminated by a pair of countergoing laser beams of the same frequency ω_p, as shown in Fig. 1. The light fields are detuned far below the atomic transition frequency ω_a. Absorption and stimulated emission of photons dominate the scattering processes and scatter the atoms into discrete momentum states.

The Hamiltonian which describes the coupling of the Bose condensate with the electromagnetic fields can be written

$$\mathscr{H} = \int d^3r\,\hat{\Psi}^\dagger(r)\left[-\frac{\hbar^2}{2M}\nabla^2 + \hat{\mathbf{E}}(r)\cdot\hat{\mathbf{D}}(r)\right]\hat{\Psi}(r) + \frac{1}{2}\int d^3r d^3r'\,\hat{\Psi}^\dagger(r)\hat{\Psi}^\dagger(r')V_{\text{inter}}(r-r')\hat{\Psi}(r')\hat{\Psi}(r), \tag{1}$$

where $\hat{\Psi}^\dagger(r)$, $\hat{\Psi}(r)$ are the creation and annihilation field operators of the condensate, respectively. $\hat{\mathbf{E}}(r)$ is the electric field operator of the electromagnetic field, and $\hat{\mathbf{D}}(r)$ is the atomic dipole moment operator. $V_{\text{inter}}(r-r')$ is the atomic interaction potential. The BEC here is treated as an ensemble of two-level atoms condensed to their stationary ground states.

For an elongated BEC, its transverse freedom is frozen by the tightly bounded trapping potential, and thus its transverse modes are much harder to excite than the longitudinal modes. Therefore, it is reasonable to assume that the transverse freedom of the system stays in its ground state, and the atom-field system can be reduced to a quasi-one-dimensional one. When the condensate is exposed to the laser beams for a sufficiently long time, it will get Bragg diffracted by the lights. The photon absorption and emission can change the momentum of an atom in the condensate by $2n\hbar k_L$ each time ($n = 0, \pm 1, \pm 2\ldots$; k_L is the wave vector of the forward beam). This suggests that we should expand the atomic field operators onto the discrete quasi-modes $\{|p_n\rangle\}$ which correspond to the states of different diffraction orders with central momenta at $\{2n\hbar k_L\}$. Even though such an expansion is not rigorously complete mathematically, it is helpful for us to

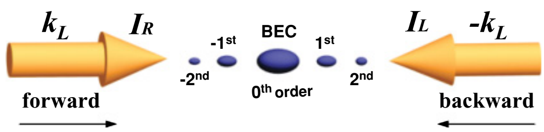

FIG. 1. (Color online) Schematic of the experimental setup. An elongated condensate is illuminated by a pair of oppositely directed laser beams with the same frequency.

extract the main information from the matter-wave diffraction. With this simplification, the atomic field operator can be expanded as

$$\hat{\Psi}(x) = \sum_n \langle x|p_n\rangle \hat{b}_n, \tag{2}$$

where we approximate $\langle x|p_n\rangle$ with plane wave $\langle x|p_n\rangle = \frac{1}{\sqrt{V}}\exp[(i/\hbar)p_n\cdot x]$, with $p_n = 2n\hbar k_L$, and where V is the volume of the system. The particle number of the atoms is normalized to unity.

Plugging Eq. (2) into Eq. (1), we arrive at the effective Hamiltonian

$$\hat{H} = \sum \hbar\omega_k b_k^\dagger b_k + J\sum\{b_{k+1}^\dagger b_k + b_k^\dagger b_{k+1}\} + \frac{1}{2}\sum_{k_1,k_2,q} U(q) b_{k_1+q}^\dagger b_{k_2-q}^\dagger b_{k_1} b_{k_2}, \tag{3}$$

where $b_k^\dagger$ and b_k are the atomic creation and annihilation operators for the kth momentum state, respectively; $\hbar\omega_m = 2(\hbar m k_L)^2/M$ is the energy of the corresponding state, $J = -\hbar G = -\hbar\Omega_1\Omega_2/\triangle$ is the hopping energy for one atom getting scattered from one momentum state to the adjacent state, and $G = \Omega_1\Omega_2/\triangle$ is the two-photon Rabi frequency with Ω_1,Ω_2 being the Rabi frequencies of the forward and backward beams, respectively. Here we do not impose strict constraints on the system that the pump fields should take exactly equal intensity. A slight difference in the laser powers can be permitted so that the uneven superradiance-induced asymmetry in the matter-wave scattering would be negligible. The excited internal state is adiabatically eliminated, which is justified for large detuning, $\triangle = \omega_a - \omega_p$. $U(q)$ is the Fourier transform of the interatomic potential, and the atomic interaction conserves the total momentum of the particles participating in collision. The indexes k,q,k_1,k_2 run over all the discrete momentum states.

III. MEAN-FIELD EFFECT OF ATOMIC INTERACTION

In a real matter-wave diffraction experiment, the mean-field effect resulting from the atomic interaction would influence the momentum distribution of the matter wave among different diffraction orders. Here we treat the atomic ensembles composed of atoms within different diffraction orders as the basic elements interacting with each other, neglecting their structural details. This is a great simplification; and such treatment can help us in understanding the dynamical behavior of diffracted matter waves.

Utilizing the simplicity we introduced in the previous section, we can further simplify the last interaction term in Eq. (3) by rearranging the operators in the product and transforming one pair of creation and annihilation operators $b_i^\dagger b_j$ via introducing the fluctuation term $\triangle(b_i^\dagger b_j) = b_i^\dagger b_j - \langle b_i^\dagger b_j\rangle$. Then we are left with

$$\begin{aligned}\sum_{k_1,k_2,q} & U(q) b_{k_1+q}^\dagger b_{k_2-q}^\dagger b_{k_1} b_{k_2} \\ &\approx \sum_{k_1,k_2,q} U(q)\big[\triangle\big(b_{k_1+q}^\dagger b_{k_2}\big)\big\langle b_{k_2-q}^\dagger b_{k_1}\big\rangle \\ &\quad + \big\langle b_{k_1+q}^\dagger b_{k_2}\big\rangle\triangle\big(b_{k_2-q}^\dagger b_{k_1}\big) + \big\langle b_{k_1+q}^\dagger b_{k_1}\big\rangle\triangle\big(b_{k_2-q}^\dagger b_{k_2}\big) \\ &\quad + \triangle\big(b_{k_1+q}^\dagger b_{k_1}\big)\big\langle b_{k_2-q}^\dagger b_{k_2}\big\rangle\big],\end{aligned} \tag{4}$$

in which we have omitted the c-number products on the right side and have neglected high-order quantum fluctuations. The shorthand $\langle\ldots\rangle$ signifies taking the average over a state of the atom-field system at some time. Thus we have $\langle b_i^\dagger b_j\rangle = \langle b_i^\dagger b_i\rangle\delta_{i,j}$. Plugging into Eq. (4) and excluding the self-interacting term, the interacting term [Eq. (4)] can be reduced to $\sum_{k,q\neq 0}\{U(q)+U(0)\}\langle n_{k+q}\rangle n_k$, with $n_k = b_k^\dagger b_k$ being the atomic number operator of the kth momentum state. Here we have assumed that the interatomic potential is an even function of q. This is always true, if the atomic interaction is isotropic in space, for example, if the Coulomb potential $U_{\text{Coulomb}}(q) = 4\pi e^2/q^2$ and the δ potential $\delta(q) =$ constant.

Finally, we arrive at an effective Hamiltonian

$$\hat{H} = \sum \hbar\tilde{\omega} b_k^\dagger b_k + J\sum b_{k+1}^\dagger b_k + J\sum b_k^\dagger b_{k+1}. \tag{5}$$

The energy spectrum of the atoms is modified by the atomic interaction with $\hbar\tilde{\omega}_k = \hbar\omega_k + \sum_{q\neq 0}[U(q)+U(0)]\langle n_{k+q}\rangle$. Changing the Hamiltonian [Eq. (5)] into the interaction picture, and using the probability amplitude method mentioned in [19], the evolution equation of the probability magnitude for the nth diffraction order of the matter wave can be obtained:

$$\dot{\Psi}_n(t) = -i[Ge^{-i\tilde{\delta}_- t}\Psi_{n-1} + Ge^{-i\tilde{\delta}_+ t}\Psi_{n+1}], \tag{6}$$

where the operators $\{b_k\}$ are replaced with c numbers $\{\Psi_k\}$, $\tilde{\delta}_\pm = \tilde{\omega}_{n\pm 1} - \tilde{\omega}_n = \pm 16\pi\nu_r(n\pm\frac{1}{2}) + 1/\hbar\sum_{q\neq 0}[U(q)+U(0)][\langle n_{n+q\pm 1}\rangle - \langle n_{n+q}\rangle]$; $\nu_r = \hbar k_L^2/(4\pi M)$ is the atomic one-photon recoil frequency, and Ψ_n is a function of the pulse duration t.

To see in detail how the atomic interaction can affect the matter-wave diffraction, we introduce the typical contact interaction

$$U(q_n) = U_0 = \frac{4\pi\hbar^2\rho a}{M}. \tag{7}$$

This is the Fourier transform of a short-range interaction taking the form of a Dirac δ function in space; ρ is the particle density of the condensate. It is directly related to the strength of the atomic interaction, where a is the s-wave scattering length.

If the density of the condensate is very low, the atomic interaction can be neglected according to Eq. (7). Taking $U(q) = 0$, the evolution equation [Eq. (6)] for the nth diffraction order would then read as follows:

$$\dot{\Psi}_n(t) = -i[Ge^{-i\delta_- t}\Psi_{n-1} + Ge^{-i\delta_+ t}\Psi_{n+1}], \tag{8}$$

where $\delta_\pm = \pm 16\pi\nu_r(n\pm 1/2)$. This equation is used by Li *et al.* [18] in studying matter-wave self-imaging for the noninteracting BEC.

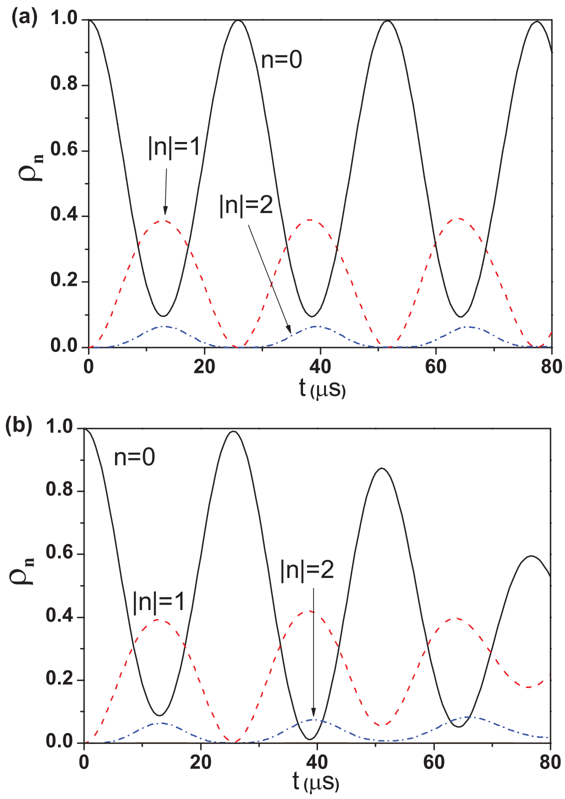

FIG. 2. (Color online) Probabilities of nth-order diffraction as a function of pulse duration for the interacting and noninteracting BECs. The two-photon Rabi frequency of the pulse fields is chosen as $G/\nu_r = 23$; $\nu_r = 3.77kHz$. (a) Bragg diffraction for the noninteracting BEC. Black solid curve, $n = 0$; red dashed curve, $n = \pm 1$; blue dash-dotted curve, $n = \pm 2$; orange solid curve, $n = \pm 3$ (almost not visible). (b) Matter-wave diffraction with the presence of repulsive atomic interaction. The interaction strength is chosen as $U_0/(\hbar\nu_r) = 1.0$. [Line styles are same as those in (a)].

In the Bragg regime with long pulse durations, the atomic center-of-mass motion contributes greatly to the matter-wave diffraction. However, in general, Eq. (6) cannot be solved analytically. Figure 2 shows the numerical simulations for the probability amplitudes of the first few order diffractions for the interacting and noninteracting BECs. In Fig. 2(a) (the noninteracting case), the momentum distribution of the diffracted matter-wave exhibits a steady oscillation in the time domain. At $t \approx 26$ μs, amplitudes of all the nonzero-th orders get to their minima simultaneously; a full matter-wave self-imaging is achieved. Figure 2(b) shows the momentum distribution with a repulsive atomic interaction at present, $U_0/(\hbar\nu_r) = 1.0$. The first matter-wave self-imaging is still observable. However, the collective momentum oscillation is dampened, implying that the atomic coherence within the condensate is weakened by the atomic interaction. This agrees with our understanding that interparticle interaction tends to smear out the collective fine orders in the many-body systems.

With the interaction strength increasing, the collective momentum oscillation of the system dies out more quickly. Figure 3(a) shows the momentum distribution of the first few order diffractions at a higher repulsive interaction strength, $U_0/(\hbar\nu_r) = 4.0$. It can be seen that matter-wave self-imaging is abandoned. In Figs. 3(b) and 3(c), we show the momentum distribution of the scattered matter wave with attractive interactions. Like the cases for repulsive interactions, the collective momentum oscillation of the diffracted matter wave is suppressed. Furthermore, comparing the results in Figs. 3(a) and 3(c), we find that the repulsive interaction tends to make the $n = 0, \pm 1$ diffraction orders become equally occupied, while the attractive interaction would establish an equilibrium in which the atomic numbers on $n = 0, \pm 1$ orders deviate greatly from each other. We will dive into the physics of such interesting phenomena in the following section.

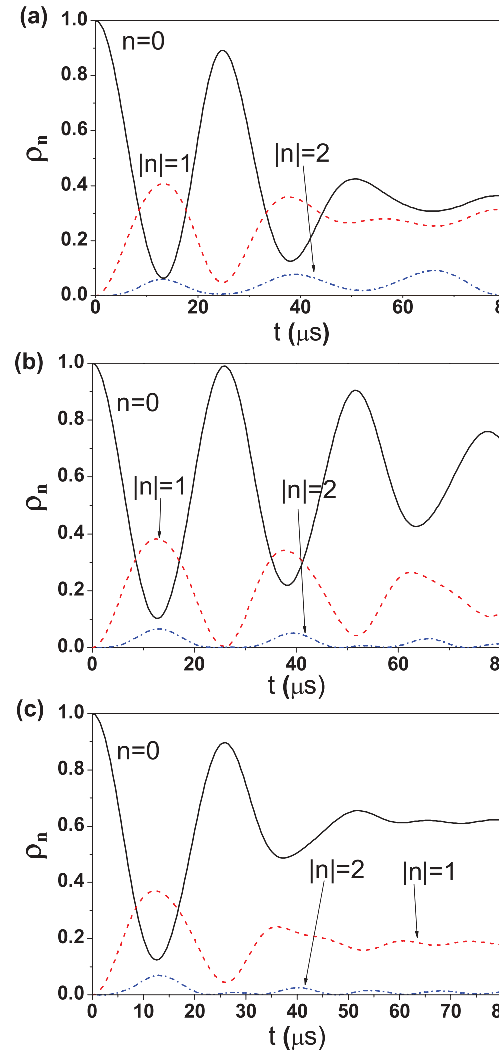

FIG. 3. (Color online) Probabilities of nth-order diffraction as a function of pulse duration for interacting BECs. The two-photon Rabi frequency is the same as that in Fig. 2. (a) Matter-wave diffraction with repulsive atomic interaction: $U_0/(\hbar\nu_r) = 4.0$. (b) Matter-wave diffraction with attractive interaction: $U_0/(\hbar\nu_r) = -1.0$. (c) Matter-wave diffraction with attractive interaction: $U_0/(\hbar\nu_r) = -4.0$. [Line styles are the same as those in Fig. 2(a)].

IV. DISCUSSION

Introducing the reduced interaction strength $R_{in} = U_0/(\hbar\nu_r)$, we define the contrast ratio of nth-order diffraction as $C_n(t) = |(\rho_n^{\max} - \rho_n^{\min})/(\rho_n^{\max} + \rho_n^{\min})|$, with $\rho_n^{\max}, \rho_n^{\min}$

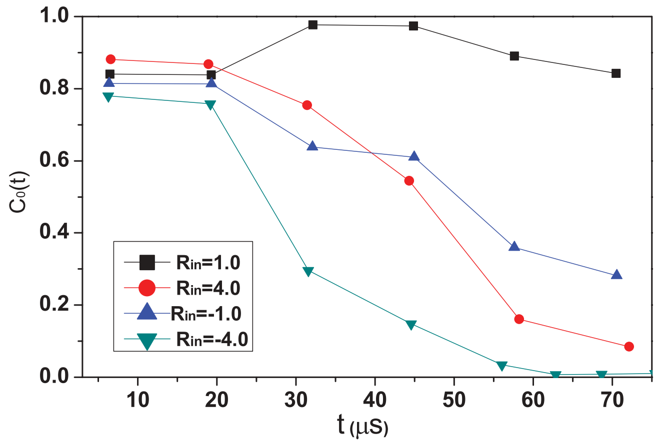

FIG. 4. (Color online) Contrast ratios of 0th-order diffraction for different kinds of interactions. The reduced interaction strength $R_{in} = U_0/(\hbar\nu_r)$ is chosen to be ± 1.0 and ± 4.0, respectively. For $R_{in} > 0$, the interaction is repulsive; for $R_{in} < 0$, the interaction is attractive.

being the two adjacent probability extrema in the time domain. Figure 4 shows the contrast ratio for the zeroth-order diffraction at different interaction strengths. We find that the contrast ratio for the repulsive interaction is larger and decreases more slowly than that for the attractive one of the same strength, whereas in the noninteracting cases, the atoms in the condensate are in perfect coherence with each other and the contrast ratio for the zeroth order remains at a high constant value [this is implied in Fig. 2(a)]. Thus the repulsive interaction can maintain the atomic coherence longer than the attractive one.

The physics behind the phenomena mentioned previously can be tracked by analyzing the dynamical behaviors of the atoms. When the matter wave is getting scattered, the atoms within different diffraction orders will feel different forces, depending on the particle number distribution among different momentum states. The atomic occupation difference at equilibrium, as shown in Figs. 3(a) and 3(c) for the repulsive and attractive interactions, respectively, can be understood in such a way: In the matter-wave diffraction processes, the hopping term and the two-body interacting term in the Hamiltonian [Eq. (3)] compete with each other. We treat the atoms in each momentum state as a whole, so by Newton's third law, the different atomic ensembles would mutually exert force on each other. For the repulsive interaction, atoms exert repulsive force on each another. The condensate is initially prepared in the stationary state (i.e., zeroth momentum state); once there is a single atom getting scattered to the $+1$th or -1th momentum state, it will feel a greater repulsive force, $F_{01}/N_{\pm 1}$, than the force felt by a single atom in the original stationary state, F_{10}/N_0, because at this time, the particle number in the zeroth momentum state, N_0, is much greater than that in the ± 1th momentum states, $N_{\pm 1}$; here F_{ij} is the mutual force exerted by the ensemble of atoms from the ith diffraction order to the ensemble of atoms in the jth order, $|F_{ij}| = |F_{ji}|$. Such a force will prevent the hopped atom from going back to its original state. This situation will not change until the $0, \pm 1$th momentum states get equally occupied; at that time, the repulsive force each particle feels in these states is nearly the same; dynamical balance is achieved. This is the reason why the atoms with repulsive interaction tend to equally occupy the $0, \pm 1$th diffraction orders at equilibrium, whereas for the attractive interaction, the situation is similar to that in the repulsive case, except that the forces between the atoms are attractive. When a small number of atoms are scattered from the zeroth momentum state to the ± 1th diffraction orders, each will feel a force trying to pull them back to their original states; this force is greater than for a single atom staying in the zeroth momentum state, so the hopped atoms tend to go back to their original stationary states. Thus, at last, the hopping mechanism and the pull-back effect resulting from the attractive atomic interaction will achieve a dynamical balance at which the zeroth diffraction order is better occupied. Thus we find that compared to the attractive interaction, the repulsive interaction is less in conflict with the atomic transportation between different diffraction orders. Therefore the strength of the momentum oscillation is less weakened by the repulsive force, which leads to a higher contrast ratio (i.e., a better atomic coherence).

The detection of the mean-field effect from the atomic interaction can be broken down into the following stages: First, one can prepare a BEC of ^{87}Rb atoms in the $|1,1\rangle$ state trapped in an optical dipole trap, using a similar method adopted by Marte *et al.* [20]. Second, the BEC is illuminated by a pair of far-red-detuned laser beams from the same source as shown in Fig. 1 ($\Delta = -1.5$ GHz; here we use the D_2 line for the proposed experiment, and the laser power can be chosen as $I_R = I_L \approx 17.6$ mW/cm^2, $\nu_r = 3.77$ kHz, $\lambda = 780$ nm). One can adjust the strength of the atomic interaction by changing the particle density of the condensate. Before the pulse fields are turned on, let the condensate expand for different durations (e.g., 1 ms, 2 ms, ...; the particle number density can be measured experimentally at the same condition for each time of matter-wave expansion). Another way to change the atomic interaction strength is to use the Feshbach resonance [21], in which a magnetic field is applied to the system to alter the atomic s-wave scattering length. The broadest resonance for the ^{87}Rb atoms in the $|1,1\rangle$ state is centered around $B_0 = 1007$ G, with a width $\Delta_B = 170$ mG. At a typical particle number density of 10^{14}/cm^3 [22], when $B = B_0 - 69.2$ mG, the corresponding interaction strength is $R_{in} \approx 4\nu_r$; when $B = B_0 + 38.1$ mG, the corresponding interaction strength is $R_{in} = -4\nu_r$. The applied magnetic field should not be too close to the resonance point to avoid the interaction strength fluctuation. All through the experiment, the magnetic field should be ramped adiabatically, except that when the interaction is changed from repulsive to attractive, the magnetic field should be scanned quickly through the resonance point to avoid giant atomic loss. After the pulse field has been turned on for different durations $(0.5, 1.0, 1.5, \ldots\, \mu$s), the trapping field and the applied magnetic field should be turned off quickly to let the BEC freely expand for about 30 ms. The momentum distribution of the diffracted matter wave can be measured by optical imaging. Repeating these steps, full information on the matter-wave momentum distribution in the time domain can be obtained.

V. CONCLUSION

We have investigated the Bragg diffraction of an elongated BEC exposed to a pair of laser beams from the same source. It is found that the full-amplitude momentum oscillation of the diffracted matter wave in a time domain, which should be established within a noninteracting BEC, would be dampened by the atomic interactions. The atomic interaction sensitively modifies the atomic coherence of the condensate, which provides us with a brilliant laboratory with tunable atomic coherence for further investigations of light-atom systems. It is further found that the repulsive and attractive interactions would establish surprisingly distinguishable equilibria for the atomic occupations among different diffraction orders; the repulsive interaction tends to make the $n = 0, \pm 1$ diffraction orders become equally occupied, while the attractive interaction would establish an equilibrium at which the atomic numbers at $n = 0, \pm 1$ orders deviate from each other. This interesting phenomenon can help us in enabling investigations in quantum many-body systems. The notions and methods demonstrated here have great potential in the fields of atom optics, atomic interferometers, and precision measurements. They can be extended to investigate other physical systems with complicated interactions.

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