

Domain-wall solutions of spinor Bose-Einstein condensates in an optical lattice

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(Received 25 April 2009; published 28 January 2010)

We studied the static and dynamic domain-wall solutions of spinor Bose-Einstein condensates trapped in an optical lattice. The single and double domain-wall solutions are constructed analytically. Our results show that the magnetic field and light-induced dipolar interactions play important roles for both the formation of different domain walls and the adjusting of domain-wall width and velocity. Moreover, these interactions can drive the motion of the domain wall in Bose ferromagnet systems similar to the way the external magnetic field or the spin-polarized current drives it in fermion ferromagnets.

DOI: [10.1103/PhysRevA.81.015602](https://doi.org/10.1103/PhysRevA.81.015602)

PACS number(s): 03.75.Mn, 03.75.Lm, 05.30.Jp, 75.60.Ch

During the past several decades, the dynamics of spatial domain walls have attracted more attention in ferromagnetism [1,2]. The conventional ferromagnet is usually composed of fermions which contribute the main site-to-site exchange interaction owing to the direct Coulomb interaction among electrons and the Pauli exclusion principle. This exchange interaction causes spin waves to become unstable, and their developing instability brings about the appearance of spatial domain walls or magnetic solitons [3]. The contribution of the magnetic dipole-dipole interaction to domain-wall formation is usually neglected in practice because it is typically several orders of magnitude weaker than the exchange coupling in a fermion ferromagnet.

However, the description of magnetism composed of bosons is not well explored. Fortunately, a typical Bose system was realized in cold spinor ⁸⁷Rb gases [4,5] and ²³Na gases [6], which provided a totally new environment to understand magnetism comprehensively. The ferromagnetic spinor Bose gases have attracted much theoretical and experimental interest [7–16], and some general properties, as in conventional fermion ferromagnets, have been observed, such as—spin domains and textures [7], spontaneous symmetry-broken ground state [8], and normal spin-wave excitation spectra [9]. Especially, the spontaneous symmetry breaking in ⁸⁷Rb spinor condensates [15] clearly shows ferromagnetic domains and domain walls in a Bose ferromagnet. Recently, the ferromagnetism in a Fermi gas has become a subject of interest [17] in the field of ultracold atoms.

Spinor Bose-Einstein condensates (BECs) trapped in an optical potential have internal degrees of freedom due to the hyperfine spin of the atoms which liberate a rich variety of phenomena. When the potential valley is so deep that the individual sites are mutually independent, spinor BECs at each lattice site behave like spin magnets and can interact with each other through both the magnetic and the light-induced dipole-dipole interactions, which are no longer negligible due to the large number of atoms at each lattice site. These site-to-site dipolar interactions can cause ferromagnetic phase

transition [9], leading to a “macroscopic” magnetization of the condensate array and spin-wave-like excitation [9,10] and magnetic solitons [18,19], analogous to ferromagnetism in solid-state physics, but occur with bosons instead of fermions. Also, spinor BECs in an optical lattice is a typical physical realization of the Salerno model [20].

In this article, we explore the domain-wall solutions of spinor BECs trapped in an optical lattice and the roles of the magnetic and light-induced dipole-dipole interactions for the type of domain-wall solutions and domain-wall width and velocity.

We consider $F = 1$ spinor condensates trapped in a one-dimensional optical lattice formed by two polarized laser beams counterpropagating along the y axis with the two laser beams detuned far from atomic resonance. Under the tight-binding approximation, the Hamiltonian takes the form [7–10]

$$\hat{H} = \sum_n \left[\lambda'_a \hat{S}_n^2 - \gamma_B \hat{S}_n \cdot \mathbf{B} - \sum_{l \neq n} J_{nl}^{\text{iso}} \hat{S}_n \cdot \hat{S}_l - \sum_{l \neq n} J_{nl}^{\text{tran}} (\hat{S}_n^+ \hat{S}_l^- + \hat{S}_n^- \hat{S}_l^+) \right], \quad (1)$$

where \hat{S}_n is the n th collective spin operator, defined as $\hat{S}_n = \hat{a}_\alpha^\dagger(n) \mathbf{F}_{\alpha\nu} \hat{a}_\nu(n)$, with \hat{a} being the annihilation operator and \mathbf{F} being the vector operator for the hyperfine spin of an atom. The first term in the Hamiltonian results from the spin-dependent interatomic collisions at a given lattice site, with $\lambda'_a = (1/2)\lambda_a \int d^3r |\phi_n(\mathbf{r})|^4$, where λ_a is proportional to the difference between the s -wave scattering lengths in the triplet and singlet channels [8] and $\phi_n(\mathbf{r})$ is the ground-state wave function for n th site. The direction of the magnetic field \mathbf{B} is along the z axis and $\gamma_B = \mu_B g_F$ is the gyromagnetic ratio, with μ_B being the Bohr magneton and g_F the Landé factor. The last two terms describe the site-to-site spin coupling induced by both the static magnetic and the light-induced dipole-dipole interactions [10], with the form $J_{nl}^{\text{iso}} = J'_{nl} + J''_{nl}$, where

$J_{nl} = \mu_0 \gamma_B^2 / (16\pi \hbar^2) \int d\mathbf{r} d\mathbf{r}' |\phi_n(\mathbf{r})|^2 |\phi_l(\mathbf{r}-\mathbf{r}')|^2 / |\mathbf{r}'|^3$, $J_{nl}'' = -3\mu_0 \gamma_B^2 / (16\pi \hbar^2) \int d\mathbf{r} d\mathbf{r}' |\phi_n(\mathbf{r})|^2 |\phi_l(\mathbf{r}-\mathbf{r}')|^2 y'^2 / |\mathbf{r}'|^5$, $J_{nl}^{\text{tran}} = \gamma U_0 / (24\Delta \hbar^2 k_L^3) \int d\mathbf{r} d\mathbf{r}' f_c(\mathbf{r}') e^{-(r_{\perp}^2 - |\mathbf{r}_{\perp} - \mathbf{r}'_{\perp}|^2) / W_L^2} \cos(k_L y - k_L y') \cos(k_L y) \mathbf{e}_{\pm 1} \cdot \mathbf{W}(\mathbf{r}') \cdot \mathbf{e}_{\pm 1} |\phi_l(\mathbf{r}-\mathbf{r}')|^2 |\phi_n(\mathbf{r})|^2 + 3J_{nl}'' / 2$. Here the cutoff function $f_c(\mathbf{r}) = e^{-r^2/L_c^2}$ describes the effective interaction range of the light-induced dipole-dipole interaction, with L_c being the coherence length associated with different decoherence mechanisms [21] and Δ being the detuned frequency. The wave number $k_L = 2\pi/\lambda_L$, the transverse coordinate $r_{\perp} = \sqrt{x^2 + z^2}$, W_L is the width of the lattice laser beams, and U_0 denotes the depth of optical lattice potential. The $\mathbf{e}_{\pm 1}$ are unit vectors in the spherical harmonic basis, and the form of tensor $\mathbf{W}(\mathbf{r})$ can be found in Ref. [10]. Equation (1) shows that the static magnetic and the light-induced dipole-dipole interaction can lead to the isotropic spin coupling denoted by J_{nl}^{iso} and anisotropic spin coupling in the transverse direction denoted by J_{nl}^{tran} .

From Hamiltonian (1) we can derive the Heisenberg equation of motion at the k th site for the spin (i.e., $i\hbar \partial \hat{S}_k / \partial t = [\hat{S}_k, \hat{H}]$). When the optical lattice is infinitely long and in the ultralow temperatures for condensation, the operator can be treated as a classical vector, $\hat{\mathbf{S}}_k \rightarrow \mathbf{S}(y, t)$. Here we assume all nearest-neighbor interactions are the same, which is a good approximation in a one-dimensional optical lattice [22]. Then we get the effective Landau-Lifshitz equation [18],

$$\frac{\partial \mathbf{S}}{\partial t} = \frac{1}{\hbar} \mathbf{S} \times \left[2Jd_0^2 \left(\frac{\partial^2 \mathbf{S}}{\partial y^2} - \frac{4J^{\text{tran}}}{J} S^z \mathbf{e}_z \right) + \gamma \mathbf{B} \mathbf{e}_z \right], \quad (2)$$

where $J = 2J^{\text{tran}} + J^{\text{iso}}$ and $d_0 = \lambda_L/2$ is the lattice constant. In a rotating frame around the z axis with angular frequency $\gamma_B B/\hbar$, the spin vector \mathbf{S} is related with the original one by the transformation $S_{\pm} \equiv S^x \pm iS^y = S'_{\pm} e^{-i\gamma_B B t/\hbar}$. Thus, Eq. (2) becomes

$$\frac{\partial \mathbf{S}}{\partial t} = \mathbf{S} \times \left(\frac{\partial^2 \mathbf{S}}{\partial y^2} - \beta S^z \mathbf{e}_z \right), \quad (3)$$

where the superscript ($'$) is omitted for pithiness, and the time t and coordinate y is in units $t_0 = \hbar/(2J)$ and d_0 , respectively. The parameter $\beta \equiv 4J^{\text{tran}}/J$ can be controlled by tuning the lattice laser beams and the shape of the condensate at each lattice site. Such a character makes the lattice atomic spin chain an ideal tool to study a diversity of spin-related phenomena.

Static domain-wall solutions. We first seek the single domain-wall solution in the form

$$S^x = \frac{\cos \eta_0''}{\cosh \Theta_0}, \quad S^y = \frac{\sin \eta_0''}{\cosh \Theta_0}, \quad S^z = \tanh \Theta_0, \quad (4)$$

where $\Theta_0 = k_0 y - \omega_0 t + \eta_0'$ with η_0' , η_0'' being two real parameters and k_0 , ω_0 to be determined. Substituting Eq. (4) into Eq. (3) we find that $\omega_0 = 0$ and $k_0 = \sqrt{-\beta}$, and solution (4), in fact, denotes the static domain wall. This result implies the parameter $\beta < 0$, which can be realized by a blue-detuned lattice ($\Delta < 0$), where the condensed atoms are trapped at the standing-wave nodes and the laser intensity is approximately zero. As a result, the light-induced dipole-dipole interactions are very small and the spin coupling is governed mainly by the static magnetic dipole-dipole interaction, which implies $|\beta| \ll 1$. In this case the domain-wall width, defined by $1/k_0$, can be adjusted mainly by J^{iso} . Under the condition

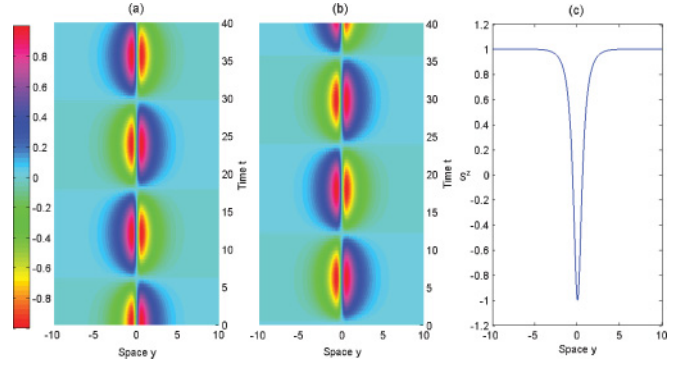


FIG. 1. (Color online) The illustration of the double domain solutions (a) S^x , (b) S^y , and (c) S^z in Eq. (5) with condition $\beta = -0.2$ and $k_1 = 0.3$. The two components S^x and S^y possess the double peak located at $\sinh \Theta_1 = \pm 1/\delta_1$.

$J^{\text{tran}}/J^{\text{iso}} \approx -0.25$, the domain-wall width is about $3d_0$, with d_0 being the lattice constant.

Equation (3) has a norm form of Landau-Lifshitz type for a spin chain with an anisotropy. It can be solved by an inverse scattering method where a couple of Lax equations are introduced for constructing the analytical solutions. In terms of our earlier results [23], we get two static double domain-wall solutions. The first has the form

$$\begin{aligned} S^x &= -2\delta_1 (\cos \Phi_1 \sinh \Theta_1) / \Delta_1, \\ S^y &= -2\delta_1 (\sin \Phi_1 \sinh \Theta_1) / \Delta_1, \\ S^z &= 1 - 2/\Delta_1, \end{aligned} \quad (5)$$

where $\Delta_1 = 1 + \delta_1^2 \sinh^2 \Theta_1$, $\Theta_1 = [\beta/(8k_1) - 2k_1]y$, $\Phi_1 = [2k_1 + \beta/(8k_1)]^2 t$, $\delta_1 = (16k_1^2 + \beta)/(16k_1^2 - \beta)$, and k_1 is a real parameter. The illustration of the domain-wall solution in Eq. (5) is shown in Fig. 1. From Fig. 1 and Eq. (5) we see that the two components S^x and S^y precess around the component S^z with the frequency $[2k_1 + \beta/(8k_1)]^2$, and the z component of spin vector S^z is conservative. The component S^x (or S^y) possesses the double peak located at $\sinh \Theta_1 = \pm 1/\delta_1$, which varies with time periodically, as shown in Fig. 1. The width of double domain wall is $1/[\beta/(8k_1) - 2k_1]$, which is in inverse proportion to the parameter β , while the maximum absolute value of valley for S^z is constant.

The other solution can be written as

$$\begin{aligned} S^x &= 2[\cot \varphi \sin(\varphi - \Phi_2) \cosh \Theta_1'] / \Delta_1', \\ S^y &= 2[\cot \varphi \cos(\varphi - \Phi_2) \cosh \Theta_1'] / \Delta_1', \\ S^z &= 1 - 2/\Delta_1', \end{aligned} \quad (6)$$

where $\Delta_1' = 1 + \cot^2 \varphi \cosh^2 \Theta_1'$, $\Phi_2 = \beta(\cos^2 \varphi)t$, $\Theta_1' = -\sqrt{-\beta}(\sin \varphi)y$, and φ is a real parameter. The illustration of the domain-wall solution in Eq. (6) is shown in Fig. 2. Different from the former one, the two components S^x and S^y possess of the single peak located at $y = 0$, which also oscillates with time periodically, as shown in Fig. 2. The width of the double domain wall is $1/(\sqrt{-\beta} \sin \varphi)$, which is in inverse proportion to $\sqrt{-\beta}$. The maximum absolute value of valley for S^z is $1 - 2/(1 + \cot^2 \varphi)$, which is also independent of the parameter β . We can rewrite S^z in Eqs. (5) and (6) as $S^z = [(\delta_1^2 + 1) \tanh^2 \Theta_1 - 1] / [(\delta_1^2 - 1) \tanh^2(-\Theta_1) - 1]$ and $(\cot^2 \varphi - 1 + \tanh^2 \Theta_1') / (\cot^2 \varphi + 1 - \tanh^2 \Theta_1')$, respectively, which implies the double

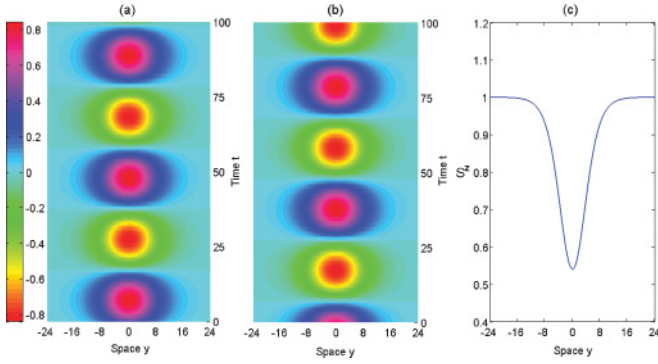


FIG. 2. (Color online) The illustration of the double domain solutions (a) S^x , (b) S^y , and (c) S^z in Eq. (6) with condition $\beta = -0.2$ and $\varphi = 0.5$. The two components S^x and S^y possess the single peak located at $y = 0$.

domain-wall solutions can be written asymptotically as a nonlinear combination of two single domain-wall solutions in Eq. (4).

For observation of the domain wall described earlier in this article, one of the possible physical realizations of a gas of dipolar BECs can be provided by electrically polarized gases of polar molecules or by applying a high dc electric field to atoms [24]. In order to induce the dipole moment of the order 0.1 D (Debye), one needs an electric field of the order of 10 V/cm and the corresponding s -wave scattering length 10–1000 Å.

Dynamic domain-wall solutions. For red-detuned lattices, the condensed atoms are trapped at the maxima of the intensity of the standing wave laser and the spin coupling is dominantly determined by the light-induced dipole-dipole interaction. In particular, the spin coupling is anisotropic in this case. By controlling the laser parameters, we may always make the light-induced dipole-dipole interaction dominate over the static magnetic dipole-dipole interaction, that is, $J^{\text{tran}} \gg J^{\text{iso}} > 0$ and $\beta \sim 2$. In this case we assume that the single moving domain-wall solution of Eq. (3) admits the form

$$S^x = \tanh \Theta_2, \quad S^y = \frac{\cos \eta_1''}{\cosh \Theta_2}, \quad S^z = \frac{\sin \eta_1''}{\cosh \Theta_2}, \quad (7)$$

where $\Theta_2 = k_2 y + \omega_2 t + \eta_1''$, with η_1'' , η_1'' are two real parameters, and k_2 and ω_2 are to be determined. Substituting Eq. (7) into Eq. (3), we obtain $\omega_2 = \beta/2 \sin(2\eta_1'')$ and $k_2 = \pm\sqrt{\beta} \sin \eta_1''$. Solution (7) shows that the dynamics of domain walls is restricted in (k_2, ω_2) space, with $|k_2|_{\text{max}} = \sqrt{\beta}$ and $|\omega_2|_{\text{max}} = \beta/2$. The width of the domain wall, defined by $1/|k_2|$, is in inverse proportion to the square root of parameter β determined mainly by light-induced dipole-dipole interaction and the angle η_1'' of the three components of pseudospin vector \mathbf{S} . The domain-wall velocity (i.e., $v = \omega_2/k_2 = \pm\sqrt{\beta} \cos \eta_1''$) is dependent on the parameters β and η_1'' . When $\eta_1'' = n\pi$, $n = 0, 1, 2, \dots$, the domain-wall velocity attains its maximum value $v_{\text{max}} = \sqrt{\beta}$. When $\eta_1'' = (n + 1/2)\pi$, $n = 0, 1, 2, \dots$, the solution in Eq. (7) represents the static domain-wall solution of Eq. (3) similar the case of $\beta < 0$. It is interesting to estimate the velocity of the domain wall. As an example, we consider the $F = 1$ electronic ground state of ^{87}Rb with Landé factor $g_F = -1/2$

and gyromagnetic ratio $\gamma_B = -\mu_B/2$. As from Ref. [9], we estimate J^{iso} is about $1.1 \times 10^{-36}\text{J}$ with $\lambda_L = 852\text{ nm}$. Assuming that $J^{\text{tran}} = 10J^{\text{iso}}$, we have $\beta \approx 1.9$ and the maximum domain-wall velocity is about $v = \sqrt{\beta}d_0/t_0 \approx 130\text{ nm/s}$. For chromium atoms, it has a magnetic moment of $6\mu_B$ and the corresponding domain-wall velocity is about tens of domain-wall velocity of ^{87}Rb with the same assumption.

The results described above in this article show that the domain-wall velocity can be controlled by the parameter β resulting from the external magnetic and light field-induced dipole-dipole interaction and the direction of pseudospin vector \mathbf{S} . In a fermion ferromagnet, a magnetic domain wall is a spatially localized configuration of magnetization in which the direction of magnetic moments gradually inverses. When a spin-polarized electric current flows through a domain wall, the spin-polarization of conduction electrons can transfer spin momentum to the local magnetization, thereby applying a spin-transfer torque, which manipulates the motion of the domain wall [25,26] similar to that seen with the application of an external magnetic field [27]. It is interesting that our domain-wall solution in Eq. (7) shows the same properties as those in a fermion ferromagnet; that is, for red-detuned lattices the light-induced dipolar interactions can be seen as the external force to drive the motion of a domain wall, and it has potential benefit for the research of domain-wall motion in the Bose ferromagnet systems.

The dynamic double domain-wall solution can be constructed by employing the Hirota method [28], which is an effective straightforward technique to solve the nonlinear equations. Here we only mention the main idea of the Hirota method briefly. First, it applies a direct transformation to the nonlinear equation. Then, by means of some skillful bilinear operators, the nonlinear equation can be decoupled into a series of equations. With some reasonable assumptions, the exact solutions can be constructed effectively. Performing the normal procedure in Ref. [28], we get the dynamic double domain-wall solution as

$$\begin{aligned} S^x &= [e^\theta \cosh^2(k_2' y) - \sinh^2(\omega_2' t) - \cos^2 \kappa] / \Delta_2, \\ S^y &= 2e^{\theta/2} \cos \kappa \cosh(k_2' y) \cosh(\omega_2' t) / \Delta_2, \\ S^z &= -2e^{\theta/2} \sin \kappa \cosh(k_2' y) \sinh(\omega_2' t) / \Delta_2, \end{aligned} \quad (8)$$

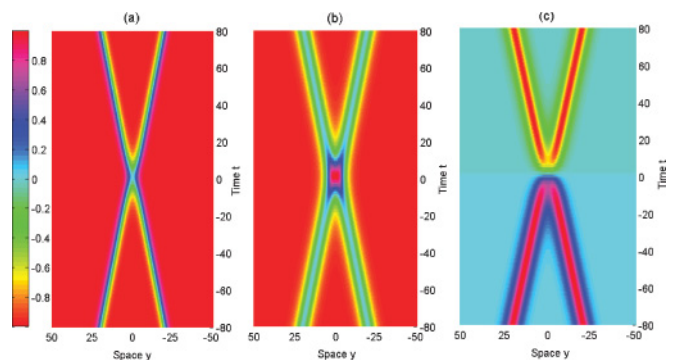


FIG. 3. (Color online) The scattering process of two moving domain walls (a) S^x , (b) S^y , and (c) S^z expressed in Eq. (8), in which two domain walls propagate with the opposite direction and the same absolute value of domain-wall velocity in the condition $\beta = 1.5$ and $\kappa = \pi/3$.

where $\Delta_2 = e^\theta \cosh^2(k'_2 y) + \sinh^2(\omega'_2 t) + \cos^2 \kappa$ with the real parameter κ , $k'_2 = \pm \sqrt{\beta} \sin \kappa$, $\omega'_2 = \beta/2 \sin(2\kappa)$, and $\theta = \ln(\cos^2 \kappa)$. In Fig. 3 we plot the evolution of double domain-wall solution \mathbf{S} in Eq. (8). From Fig. 3 we see that the double domain solution in Eq. (8) presents a general scattering process of two moving domain walls which propagate with the opposite direction and the same absolute value of domain-wall velocity $\sqrt{\beta} \cos \kappa$. Analysis reveals that there is no amplitude exchange among the components S^x for two domain walls. However, there is a small shift for the center of each domain wall during collision.

In conclusion, we have presented two kinds of domain-wall solutions of dipolar spinor BECs in an optical lattice based on an effective Hamiltonian of anisotropic pseudospin chain. In real experiments, the magnetic field and light-induced dipolar interactions are related with the detuned term, which can be controlled easily by an external magnetic and light field. Our

results show that we can control this site-to-site spin coupling of the condensates at each lattice to derive the different domain-wall solutions. Especially, since these magnetic field and light-induced dipolar interactions are highly controllable, the spinor BECs in an optical lattice as an exceedingly clean system offer a very useful tool to study spin dynamics in periodic structures and to understand ferromagnetism comprehensively.

This work was supported by NSF of China under Grants No. 10874038 and No. 10804028 and the NSF of Hebei Province of China under Grant No. A2007000006. J.-Q. Liang was supported by NSF of China under Grant No. 10775091. W. M. Liu was supported by NSFC under Grants No. 60525417, No. 10740420252, and No. 10874235 and by NKBRSCF under Grants No. 2006CB921400 and No. 2009CB930704.

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